



# **Extreme Value Distributions on Closing Quotations and Returns of Islamabad Stock Exchange**

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### **Authors' contributions**

*This work was carried out in collaboration among all authors. Author MA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors NJ and MH managed the analyses of the study. Author US managed the literature searches. All authors read and approved the final manuscript.*

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## **ABSTRACT**

This study is an experimental test done on the secondary data of banking sector of Islamabad Stock Exchange for year 2017 and applied different techniques on the given data record by using Generalized Extreme Value Distribution (GEV), Gumble Distribution (GBL), Generalized Pareto Distribution (GPD), Exponential Distribution (EXP), Gamma Distribution (GAM), Weibull Distribution (WBL) on the data of four banks Habib Bank, Allied Bank, Bank Alfalah and Askari Bank. This data is concerning the closing quotations and returns of four banks registered in Islamabad Stock Exchange. We try to fit different distributions on the data and found the best fit distribution.

We estimated the parameters of each distribution and also find the standard deviations of each distribution by using R Language and find which distribution is the best fit distribution on the basis of standard deviation distribution. We analyzed that shape wise GEV is the most suitable distribution, scale wise EXP distribution the best and location wise the best one is Gumbal distribution. This article shows that the overall GEV is the best distribution to model correctly the data.

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**Keywords:** GEV distribution; EXP distribution; GBL distribution; WBL distribution, GPD; R-language.

## 1. INTRODUCTION

From the previous three decades, the world economy has been confronting securities exchange crashes, cash emergency, the website and land bubble burst, credit crunch and saving money alarms. As a reaction, extreme value theory (EVT) gives an arrangement of instant ways to deal with chance administration investigation. In this research we have used Generalized Extreme Value Distribution (GEV), Gumble Distribution (GBL), Generalized Pareto Distribution (GPD), Exponential Distribution (EXP), Gamma Distribution (GAM), Weibull Distribution (WBL) on the data of daily closing price of Islamabad stock exchange of four banks Habib Bank, Allied Bank, Bank Alfalah and Askari Bank. An experimental test is done on the secondary data of banking sector of Islamabad Stock Exchange for year 2017 and applied different statistical techniques on the given data record to estimate the parameters of each distribution and also estimated the standard deviations (S.D) of each distribution and find which distribution is the best fit distribution on the basis of standard deviation distribution.

On the basis of their standard deviations we analyzed that shape wise GEV is the most suitable distribution, scale wise EXP distribution is the best and location wise the best fit is Gumbel distribution. This data is concerning the closing quotations and returns of four banks registered in Islamabad Stock Exchange.

For the first time the Weibull distribution is found by Fréchet in 1927. This distribution has given a name after Waloddi Weibull who introduced this distribution briefly in 1951. Ramller applied this to discuss a small particle size distribution. The shape parameter makes Weibull distribution to express in different shapes, relies upon the examined parameter estimation of the shape. These types of distributions are particularly great in displaying applications since they are parcel of edges to demonstrate numerous arrangements of information. The Weibull distribution has less confusion and has easy to express. In any case, because of the state of this parameter the Weibull to assume numerous shapes. Because of easy to express and easy to apply the Weibull Distribution has numerous and quality applications in many fields.

The gamma distribution, meant as gamma ( $\alpha, \beta$ ), is a two-parameter distribution. Because of direct

skewness, the gamma distribution is a fitting model in numerous fields of measurements in those situations where the typical distribution isn't appropriate. As a hypothesis, the gamma distribution is connected as estimation for holding up times and administration times [1]. This circulation is utilized as a part of numerous instances of environ measurements, for example, natural observing of precipitation sizes [2]. The gamma distribution is a one of the proper distribution in numerous fields of study and research [3,4]. Since this model is likewise used in flag preparing [5] and clinical preliminaries [5].

The Generalized Pareto distribution was presented by Pickands [6], and enthusiasm and appeared by Davison Smith [7], van Montfort and Witter. The applications incorporate use in the examination of extraordinary occasions, in the displaying of vast protection claims, as a disappointment time appropriation in unwavering quality investigations, and in any circumstance in which the exponential distribution may be utilized however in which some heartiness is required against heavier followed or lighter followed choices.

The Generalized Pareto distribution is the distribution of an arbitrary variable  $X$  characterized by  $\alpha(1-e^{-kY})/k$ , where  $Y$  is an irregular variable.

Gumbel distribution is a specific instance of the summed up extraordinary esteem circulation (otherwise famous as Fisher-Tippett distribution). Generally called the log-Weibull allotment and the twofold exponential scattering (a term that is on the other hand sometimes used to suggest the Laplace scattering). It is related to the Gompertz scattering: when its thickness is first reflected about the beginning stage and after that restricted to the positive half line, a Gompertz work is gained.

In probability theory and estimations, the Gumbel flow (Generalized Extreme Value transport Type-I) is used to show the apportionment of the best (or the base) of different cases of various scatterings. This apportionment might be used to address the dispersal of the most extraordinary level of a conduit in a particular year if there was a summary of most prominent characteristics for whatever length of time that ten years. It is important in anticipating the given that an unbelievable tremor, flood or other disastrous occasion will happen. The potential congruity of

the Gumbel course to address the movement of maxima relates to incredible regard theory, which demonstrates that it is presumably going to be useful if the scattering of the shrouded case data is of the normal or exponential make. Whatever is left of this article suggests the Gumbel to exhibit the scattering of the most extraordinary regard. To show the base regard, use the negative of the primary characteristics.

### 1.1 Objective of the Study

- Using extreme value distributions checked the best fit distribution on the assets price of Islamabad stock exchange data using three parameters location wise, scale wise and shape wise of each distribution using R (statistical software).

## 2. LITERATURE REVIEW

Combes and Dussauchoy [8] a similar evaluation utilizing different hypothetical distributions: Normal, LogNormal, Gamma, Gumbel, Weibull and Generalized Extreme Value (GEV). They used GEV transport in some other setting than over the top regard theory (in actuality committed to this region). From the trial distribution on brief periods (3, 6, 9 and a year), we exhibit that GEV assignment grants to precisely fit returns and opening/closing references (without focus only the direct of maxima or minima in an illustration, yet all things considered of the case) by relationship with exchange scatterings. This paper bases on the GEV distribution in the univariate case. Following a review of the composition, univariate GEV scattering is associated with a movement of consistently stock-exchange of TOTAL oil association. They demonstrate this article with the opening/closing references less the moving ordinary of the five a days back and the benefits of this association on short and medium terms (3, 6, 9, a year pushing ahead multi month).

Onen and Bagatur [9] forecasted of surge recurrence factor (K) for the Gumbel distribution utilizing quality articulation programming (GEP) and relapse demonstrate. Some desire models are presented for choosing of flood repeat factor (K). The proposed backslide illustrates (Model 4) and GEP show (Model 7) gives a snappy and practical strategy for assessing the flood repeat factor. Along these lines, Gumbel's system has been revised in such a farsighted model, to the point that one can obtain the span of a given return period for flood discharges without plan of

move to making a gander at a table. The execution of the figure models was evaluated with an illustrative case for 2, 5, 10, 20, 50, 100, 200, 250, 500 and 1000 years flood.

Chen and Ye [3] the parameters of a Weibull distribution are evaluated by most extreme probability estimation. To reduce the inclinations of the best likelihood estimators (MLEs) of two-parameter Weibull spreads, they proposed informative tendency balanced MLEs. Two other fundamental estimators of Weibull movements, scarcest squares estimators and percentiles estimators are moreover exhibited. In perspective of an examination of their displays in the entertainment consider; we immovably propose the logical tendency balanced MLEs for the parameters of Weibull transports, especially when the case assesses are nearly nothing.

Sheng and Chen [10] new estimators are developed to obtain Gamma distribution's parameter. In addition, these estimators enables bias corrections which helps in improving performance of small sample data. The graphical and analytical methods to estimate Weibull parameters is presented by Fawzan [11] and Angadi [12].

Vela and Rodríguez [13] models the distribution of loss probability and its various impacts. Chen and Zhang [14] evaluated new type of common estimators of Weibull Distribution. By simulation results, they recommend the use of analytic bias corrected MLEs to find parameters of Weibull distribution. Kwasnicka and Ciosmak [15] developed new computer system for stock market analysis. It provides advices on different policies and fundamental rules in Stock Market.

Hosking and Wallis [16] worked on estimation of Generalized Pareto distribution. For further information see [17]. Computer Simulation is used to evaluate the accuracy of confidence intervals of generalized Pareto distribution. Kotz and Nadarajah [18] the generalization of Gumbel distribution is introduced as Beta Gumbel Distribution. They study skewness and kurtosis of proposed distribution and gave its properties. Lazoglou and Anagnostopoulou [19] worked on different statistical methods that measure the occurrence of extreme rainfalls.

The generalized Pareto distribution was presented by Pickands [6], and enthusiasm for it was appeared by [7], Its applications incorporate use in the investigation of extraordinary

occasions, in the demonstrating of huge protection claims, as a disappointment time circulation in unwavering quality examinations, and in any circumstance in which the exponential appropriation may be utilized yet in which some power is required against heavier.

### 3. DATA DESCRIPTION AND METHODOLOGY

#### 3.1 Weibull Distribution

We determined in many different estimation techniques of the parameters of the two-parameter Weibull distribution, and to find two different processes to overcome the biases of maximum likelihood estimators (MLEs). Given mathematical form is the cumulative distribution function of a Weibull distribution:

$$F(x; l, m) = \begin{cases} 1 - e^{-\left(\frac{x}{m}\right)^l}, & x > 0, m > 0, \\ > 0 & x \leq 0 \end{cases}$$

and its probability density function is:

$$f(x; l, m) = \begin{cases} \left(\frac{l}{m}\right) x^{l-1} e^{-\left(\frac{x}{m}\right)^l}, & x > 0, m > 0, l \\ > 0 & x \leq 0 \end{cases}$$

The Weibull distribution is the first distribution found by Fréchet in 1927, also, this circulation has given a name after Waloddi Weibull, a Swedish mathematician who presented the weibull distribution exhaustively in 1951. Rammler used this to translate a little molecule measure appropriation. Weibull circulation isn't the main dispersion, however we can state it a group of appropriations, because of its diverse attributes like the distribution having the shape parameter. The shape parameter makes Weibull circulations to express in different shapes, relies upon the broke down parameter estimation of the shape. These sorts of conveyances are particularly great in displaying applications since they are parcel of edges to show numerous arrangements of information. The Weibull dispersion is relatively less difficulties and has easy to express. In any case, because of the state of this parameter the Weibull to assume numerous shapes. Because of every one of these attributes of effortlessly reasonable and foldability in the state of the Weibull circulation empowers it an amazing model of dispersion in tried and true use in our day by day life. The model has inclination to numerous distributional shapes by applying similarly simple distributional frame that can be occur in different other distributional families. The connection of Weibull distribution with numerous different distributions.

Shown that, it is popular that a Weibull conveyance keeps the exponential circulation (when the estimation of  $l = 1$ ) however the Rayleigh appropriation (when the estimation of  $l = 2$ ). A brief timeframe prior, Ling and Giles proposed the Rayleigh circulation as it has a predisposition inurement of the Rayleigh appropriation. The extraordinary state of the summed up outrageous esteem appropriation is the Weibull conveyance and it is spoken to by Fréchet. The pdf of Fréchet distribution is given by

$$f_{Fréchet}(x; l, m) = \begin{cases} \frac{l}{m} \left(\frac{x}{m}\right)^{l-1} e^{-\left(\frac{x}{m}\right)^l}, & x > 0, m > 0, l \\ > 0 & x \leq 0 \end{cases}$$

Actually,  $f_{Fréchet}(x; l, m) = -f_{Weibull}(x; -l, m)$ . We can also define Weibull distribution as a uniform distribution; as  $U$  is uniformly distributed when it is  $(0, 1)$ , and the random variable  $m(-\ln(U))^{\frac{1}{k}}$  is Weibull distributed with parameters  $L$  and  $m$ . There is a comprehensive use of Weibull distribution in many fields, like manufacturing, mills, analysis of datasets, efficiency in different instruments, testing of material things and assessments of environmental. This distribution was proposed and applied in an aircraft system to enquire into in or ready for use of performing consistently well by Kaltschmidt et al. Nikolaj demonstrated that this conveyance can be utilized as a part of oil contamination examination. Singh considered the Weibull distribution and worked in hydrology by utilizing this distribution. Aarset connected a Weibull distribution to test disappointment times of gadgets, and the information were additionally breaking down and altered by Lai et al. It was utilized as a part of breakdown voltage estimation by Hirose, and Fabiani utilized it to test electrical breakdown of protecting materials. It was connected to edited information by Ghitany et al. In the investigation of wind vitality, the Weibull distribution was likewise connected by Akdag et al.

#### 3.2 Gamma Distribution

We can denote gamma distribution as  $\text{gam}(a, b)$ , and it has two-parameters with probability density function (PDF)

$$f_{gam}(x) = \frac{x^{a-1}}{b^a \Gamma(a)} e^{-\frac{x}{b}}, x > 0$$

Where  $a > 0$  is the shape parameter,  $b > 0$  is the scale parameter and  $\Gamma(\cdot)$  is the gamma function. Because of the moderate skewness, the gamma

distribution is an appropriate model in many fields of statistics in those cases where the normal distribution is not suitable. For example, it is applied to model weakness and random-effects. According to queueing theory, the gamma distribution is applied as a distribution for waiting times and service times [1]. This distribution is used in many cases of environ metrics such as environmental monitoring of rainfall sizes [2]. The gamma distribution is a one of the appropriate distribution in many fields of study and research [3,4]. Because this model is also utilized in signal processing [5], and clinical trials (Wiens 1999).

### 3.3 Generalized Pareto Distribution

Pareto distribution is used in additionally the appropriation of an arbitrary variable X as Y speaks to an irregular variable with its standard exponential distribution. The Pareto circulation has likelihood dispersion work as neglected:

$$F(x) = 1 - (1 - sx/t)^{\frac{1}{s}}, \quad s \neq 0$$

$$= 1 - \exp(-x/t), \quad s = 0$$

The density function

$$f(x) = t^{-1}(1 - sx/t)^{\frac{1}{s}-1}, \quad s \neq 0$$

$$= t^{-1} \exp(-x/t), \quad s = 0$$

The range of x is  $0 < x < \infty$  for  $s \leq 0$  and  $0 < x < t/s$  for  $s > 0$ . The Pareto distribution has parameters of t as the scale parameter, and s as the shape parameter. In the special cases  $s = 0$  and  $s = 1$  it happens respectively, the exponential distribution with mean t and the uniform distribution on  $[0, t]$ ; Pareto distributions are obtained when  $s < 0$ .

### 3.4 Exponential Distribution

The exponential distribution is one of the broadly utilized persistent distributions. Usually used to show the time slipped by between occasions. We will now scientifically characterize the exponential distribution and infer its mean and standard error. At that point we will build up the instinct for the dispersion and talk about a few fascinating properties this distribution.

A continuous random variable X is said to have an exponential distribution with parameter  $\lambda > 0$ , shown as  $X \sim \text{Exponential}(\lambda)$  if its PDF is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

It is convenient to use the unit step function defined as

$$u(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

So we can write the pdf of an Exponential ( $\lambda$ ) random variable as

$$f_X(x) = \lambda e^{-\lambda x} u(x)$$

Let us find its CDF, mean and variance. For  $x > 0$ , we have

$$F_X(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}$$

So we can the CDF as

$$F_X(x) = (1 - e^{-\lambda x})u(x).$$

### 3.5 Generalized Extreme Value (Gev) Distribution

Given a time series  $(X_1, \dots, X_n)$  consisting of a sequence of independent and identically distributed (iid) random variables, the maximum  $M_{x,n} = \max(X_1, \dots, X_n)$  converges in law (weakly) to the following generalized extreme value distribution:

$$G_{ev}\left(\frac{x}{\xi}, \sigma, \mu\right) = \begin{cases} \exp\{-[1 + \xi(\frac{x-\mu}{\sigma})]^{-\frac{1}{\xi}}\} & \xi \neq 0 \\ \exp\{-\exp(-\frac{x-\mu}{\sigma})\} & \xi = 0 \end{cases} \quad 1 + \frac{x-\mu}{\sigma} > 0, \sigma > 0$$

And the density of GEV is

$$g_{ev}\left(\frac{x}{\xi}, \sigma, \mu\right) = \frac{1}{\sigma} G_{ev}\left(\frac{x}{\xi}, \sigma, \mu\right) [1 + \xi(\frac{x-\mu}{\sigma})]^{-1 + \frac{1}{\xi}} \quad 1 + \frac{x-\mu}{\sigma} > 0$$

In this distribution  $\sigma$  is representing the scale parameter,  $\mu$  is representing the location parameter. Where  $\xi$  is representing the shape parameter that shows the tail behavior. So Fréchet type tail represents to  $\xi > 0$  but Weibull type tail represented to  $\xi < 0$ . The Gumbel type is interpreted as the limit as  $\xi \rightarrow 0$  of Equation 4.1 with  $\xi = 0$ . The Fréchet distribution compares to overwhelming followed circulations and has usually been observed to be the most fitting for the substantial tail of money related information. The asymptotic dispersion of the most extreme can be assessed without making any suppositions about the idea of the first conveyance of the perceptions since the asymptotic circulation of the greatest (on the off

chance that it exists) dependably has a place with one of these three appropriations, whatever the first distribution. When the minima  $M_{x,n} = \min(X_1, \dots, X_n)$  is of concern, the distribution function is as  $G_{ev}(\frac{x}{\xi}, \sigma, \mu)$  as  $M_{x,n} = \min(X_1, \dots, X_n) = M_{x,n} = \max(X_1, \dots, X_n)$ . The reverse of circulation capacity of GEV for the maxima,  $[G_{ev}]^{(-1)}(1-p)$  speak to the quantile of  $1 - p$ , here  $p$  is the little likelihood (upper tail) as  $P(x > xp) = p$ , which  $x_p$  is known as the arrival level which have the arrival time of  $1/p$ . We can translate this as it will be watched an extraordinary more prominent than the arrival level  $x_p$ . Every  $1/p$  period is by and large or as the mean holding up time between particular extremal occasions. To deal with the hazard in the field of fund  $x_p$  is known as the Value at Risk (VAR) to express the most elevated conceivable misfortune amid a specific period  $1/p$ . For instance, a most extreme misfortune amid a time of 30 days is close to  $R_p$  is to be seen with a 5% danger of false. As indicated by this the arrival level of  $R_p$  with an arrival period 585 days ( $p = 0.0017083$ ) as understood for  $P(L(R_p) \leq 30) = 1 - ([1-p])^{30} = 0.05$ , and  $L(R_p)$  communicated for the season of first disappointment which is gathered as a Bernoulli distribution.

### 3.6 Gumbel Distribution

In likelihood hypothesis and measurements, the Gumbel distribution (Generalized Extreme Value dispersion Type-I) is utilized to demonstrate the circulation of the most extreme (or the base) of various examples of different distributions. This distribution may be utilized to speak to the distribution of the most extreme level of a stream in a specific year if there was a rundown of greatest qualities for as far back as ten years. It is valuable in foreseeing the possibility that an extraordinary tremor, surge or other catastrophic event will happen. The potential pertinence of the Gumbel distribution to speak to the appropriation of maxima identifies with extraordinary testing of hypothesis, which shows that it is probably going to be helpful if the distribution of the basic example information is of the typical or exponential compose. Whatever is left of this article alludes to the Gumbel to demonstrate the distribution of the most extreme esteem. To display the base esteem, utilize the negative of the first qualities.

The Gumbel distribution is a particular example of the summed up silly regard scattering

(generally called the Fisher-Tippett scattering). It is generally called the log-Weibull dispersal and the twofold exponential apportionment (a term that is of course all over used to imply the Laplace flow). It is related to the Gompertz movement: when its thickness is first reflected about the beginning and after that bound to the positive half line, a Gompertz work is gained.

In the idle variable arrangement of the multinomial logit exhibit fundamental in discrete choice theory the mix-ups of the dormant components take after a Gumbel scattering. This is profitable in light of the way that the qualification of two Gumbel-passed on discretionary components has an ascertained scattering.

The Gumbel scattering with zone parameter (an) and scale parameter (b) is executed in the Wolfram Language as Gumbel Distribution [alpha, beta]. It has probability thickness limit and apportionment work.

$$P(x) = \frac{1}{b} \exp \left[ -\frac{x-a}{b} - \exp \left( -\frac{x-a}{b} \right) \right]$$

$$D(x) = 1 - \exp \left[ -\exp \left( -\frac{x-a}{b} \right) \right]$$

## 4. RESULTS AND DISCUSSION

In this article we have analyzed the data of four registered bank in Islamabad Stock Market using the Generalized Extreme Value Distribution (GEV), Gumble Distribution (GBL), Generalized Pareto Distribution (GPD), Exponential Distribution (EXP), Gamma Distribution (GAM) and Weibull Distribution (WBL). We estimated the parameters of each distribution and also find the standard deviations of each distribution and find which distribution is the best fit distribution on the basis of standard deviation distribution. Distribution having the smallest value of S.E is the best fit.

This data is concerning the closing quotations and returns of four banks registered in Islamabad Stock Exchange. We try to fit different distributions on the data and analyzed the best fit. We determined that GEV is the best distribution. We analyzed the data in three ways and concluded that shape wise GEV is the most suitable distribution, scale wise EXP distribution is the best and location wise the best one is Gumbal distribution but overall GEV is the best distribution to model correctly the data.

**Table 1. Allied Bank 2017**

Distribution S.E	Parameter - I	Parameter - II	Parameter – III
GEV	Shape (0.1238393)	Scale/rate (7.283996)	Location (88.99022)
S.E	0.1254005	0.9270635	1.206489
GBL	-	Scale/rate (7.676439)	Location (89.48872)
S.E	-	0.8983158	1.161836
GPD	Shape ( -10.04509)	Scale/rate (1174.854)	-
S.E	1.305546	NaN	-
EXP	Shape	Scale/rate	Location
	-	(0.01061849 )	-
S.E	-	0.00151891	-
GAM	Shape (84.00906)	Scale/rate (0.8920622)	Location -
S.E	17.11199	0.182248	-
WBL	Shape	Scale/rate	Location
	(8.273724)	(99.19448)	-
S.E	0.8406732	1.843414	-

*The following table shows the shape wise, scale wise and location wise standard error (S.E), In shape wise comparison GPD is more appropriate distribution than others as its S.E is least which is 0.1254005, In scale wise comparison Exponential distribution is more appropriate than others as its S.E is least which 0.00151891, In location wise comparison Gumbel distribution is more appropriate than others as its S.E is least which is 1.161836*

**Table 2. Askari Bank 2017**

Distribution S.E	Parameter - I	Parameter - II	Parameter – III
GEV	Shape (0.4758544)	Scale/rate (1.308208)	Location (19.6438)
S.E	0.1874865	0.2177561	0.232455
GBL	-	Scale/rate (1.739013)	Location (20.02309)
S.E	-	0.215353	0.2619266
GPD	Shape ( -1.787793)	Scale/rate (41.29086)	-
S.E	0.1137081	NaN	-
EXP	Shape	Scale/rate	Location
	-	(0.0472654)	-
S.E	-	0.006819118	-
GAM	Shape (70.92737)	Scale/rate (3.352428)	Location -
S.E	14.44396	0.6851164	-
WBL	Shape	Scale/rate	Location
	(7.712964)	(22.37737)	-
S.E	0.7979002	0.4461521	-

*The following table shows the shape wise, scale wise and location wise standard error (S.E), In shape wise comparison GPD is a more appropriate distribution than others as its S.E is least which is 0.1137081, In scale wise comparison Exponential distribution is a more appropriate than others as its S.E is least which 0.006819118, In location wise comparison Gumbel distribution is a more appropriate than others as its S.E is least which is 0.232455*

**Table 3. Bank Alfalah 2017**

Distribution	S.E	Parameter-I	Parameter-II	Parameter-III
GEV		Shape (-0.2585817)	Scale/rate (2.02573)	Location (39.75594)
S.E	0.08785195		0.2241141	0.3216105
GBL		-	Scale/rate (1.962014)	Location (39.48114)
S.E		-	0.2084571	0.3003464
GPD		Shape (-24.67384)	Scale/rate (1022.385)	-
S.E	3.417024		NaN	-
EXP		Shape	Scale/rate (0.02469072)	Location -
S.E		-	0.003557942	-
GAM		Shape (390.7955)	Scale/rate (9.649045)	Location -
S.E	79.7371		1.970031	-
WBL		Shape (20.39839)	Scale/rate (41.47983)	Location -
S.E	0.3112296		2.134244	-

The following table shows the shape wise, scale wise and location wise standard error (S.E). In shape wise comparison GEV is a more appropriate distribution than others as its S.E is least which is 0.08785195, In scale wise comparison Exponential distribution is a more appropriate than others as its S.E is least which 0.003557942, In location wise comparison Gumbel distribution is a more appropriate than others as its S.E is least which is 0.3003464

**Table 4. Habib Bank 2017**

Distribution	S.E	Parameter-I	Parameter-II	Parameter-III
GEV		Shape (-0.7072903)	Scale/rate (50.86473)	Location (232.7484)
S.E	0.09980342		6.808803	7.858668
GBL		-	Scale/rate (45.83158)	Location (215.1644)
S.E		-	5.002405	7.032493
GPD		Shape (5.336282)	Scale/rate (1597.149)	-
S.E	0.6258883		NaN	-
EXP		Shape	Scale/rate (0.004181759)	Location -
S.E		-	0.0005664332	-
GAM		Shape (24.74098)	Scale/rate (0.1034555)	Location -
S.E	5.001373		0.02112457	-
WBL		Shape (6.730902)	Scale/rate (257.5216)	Location -
S.E	0.8288871		5.783377	-

In this table we observed the shape wise, scale wise and location wise standard error (S.E). In shape wise comparison GEV is a more appropriate distribution than others as its S.E is least which is 0.09980342, In scale wise comparison Exponential distribution is a more appropriate than others as its S.E is least which 0.0005664332, In location wise comparison Gumbel distribution is a more appropriate than others as its S.E is least which is 7.032493

## 5. CONCLUSION

We analyzed that shape wise the best distribution is generalized extreme value distribution and scale wise exponential distribution and location wise Gumbal distributions (GBL) are the most appropriate. But

according to overall analysis by using parameter estimates we concluded that generalized extreme value distribution (GEV) is the best fit for this given data set. Because it has all three parameters estimates having some values and none of the parameter is missing. For example it is giving some result in every case if we



analyzed it shape wise, scale wise and location wise.

But shape wise it has least standard deviation in each case and the best fit distribution as compared to Exponential Distribution (EXP), Gamma (GAM), Weibull (WBL), and Generalized Pareto Distribution (GPD). As we compared the standard errors (S.E) of each year and analyzed that the generalized extreme value distribution (GEV) is most suitable as compared to other distribution are used. According to generalized extreme value distribution (GEV) we can represent the given data in best way. So for this data generalized extreme value distribution (GEV) is most suitable and can define data well.

### COMPETING INTERESTS

Authors have declared that no competing interests exist.

### REFERENCES

1. Whitt W. The impact of a heavy-tailed service-time distribution upon the M/GI/s waiting-time distribution. *Queueing Systems*. 2000;36(1-3):71-87.
2. Krishnamoorthy K, Mathew T, Mukherjee S. Normal-based methods for a gamma distribution. *Technometrics*. 2008;50(1):69-78.
3. Chen P, Ye ZS. Estimation of field reliability based on aggregate lifetime data. *Technometrics*, to Appear; 2016. DOI: 10.1080/00401706.2015.1096827
4. Lawless Meeker WQ, Escobar LA. *Statistical methods for reliability data*. John Wiley & Sons: New York; 1998.
5. Vaseghi SV. *Advanced digital signal processing and noise reduction*. John Wiley & Sons: New York; 2008.
6. Pickands J. Statistical inference using extreme order statistics. *Annals of Statistics*. 1975;3:119–131.
7. Davison AC, Smith RL. Models for exceedence over high thresholds (with discussion). *Journal of the Royal Statistical Society*. 1990;52:393–442.
8. Combes C, Dussauchoy A. Generalized extreme value distribution for fitting opening/closing asset prices and returns in stock-exchange. *Operational Research*. 2006;6(1):3-26.
9. Onen F, Bagatur T. Prediction of flood frequency factor for Gumbel distribution using regression and GEP model. *Arabian Journal for Science and Engineering*. 2017;42(9):3895-3906.
10. Ye ZS, Chen N. Closed-form estimators for the gamma distribution derived from likelihood equations. *The American Statistician*. 2017;71(2):177-181.
11. Al-Fawzan MA. *Methods for estimating the parameters of the Weibull distribution*. King Abdulaziz City for Science and Technology, Saudi Arabia; 2000.
12. Angadi AB, Angadi AB, Gull KC. *International Journal of Advanced Research in Computer Science and Software Engineering*. 2013;3(6).
13. Calderon Vela A, Rodríguez G. Extreme value theory: An application to the Peruvian stock market returns; 2014.
14. Chen M, Zhang Z, Cui C. On the bias of the maximum likelihood estimators of parameters of the Weibull distribution. *Mathematical and Computational Applications*. 2017;22(1):19.
15. Kwaśnicka H, Ciosmak M. Intelligent techniques in stock analysis. In *Intelligent Information Systems*. Physica, Heidelberg. 2001;195-208.
16. Hosking JR, Wallis JR. Parameter and quantile estimation for the generalized Pareto distribution. *Technometrics*. 1987;29(3):339-349.
17. Ilie N, Hickel R. Macro-, micro-and nano-mechanical investigations on silorane and methacrylate-based composites. *Dental Materials*. 2009;25(6):810-819.
18. Nadarajah S, Kotz S. The beta Gumbel distribution. *Mathematical Problems in Engineering*. 2004;4:323-332.
19. Lazoglou G, Anagnostopoulou C. An overview of statistical methods for studying the extreme rainfalls in Mediterranean. In *Multidisciplinary Digital Publishing Institute Proceedings*. 2017;1(5):681.

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