



Determination of the Equilibrium Point According to a Temporal Model of Rehydration in a Population with Cholera in Logistic Growth

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

Cholera is an infection of the small intestine of humans caused by a gram-negative bacterium called *Vibrio cholerae*. It is spread through eating food or drinking water contaminated with faeces from an infected person. It causes rapid dehydration and general body imbalance, and can lead to death since untreated individuals suffer severely from diarrhea and vomiting. Its dynamics involves multiple interaction between the human host, the pathogen and the environment which contributes to both human to human and indirect environment to human transmission pathways. Rehydration is critical in reducing cholera death. This has been studied by other scholars but they did not consider delay in rehydration on the spread of cholera. In this paper, I formulate a mathematical model based on system of ordinary differential equation with rehydration on the spread of cholera in a logistically growing population. The existence of the steady states and the basic reproduction number is established such that disease free equilibrium point exists. Determination of endemic equilibrium shows that the model has positive points. The findings will be significant in the sense that timely rehydration should be done during cholera outbreak and will enable individuals with symptoms to seek immediate medical attention.

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1 Introduction

Cholera is an infection of small intestine caused by a gram-negative bacterium called *Vibrio cholerae*. The dynamics of cholera involve multiple interactions between the human host, the pathogen, and the environment, which contribute to both human to human and indirect environment to humans transmission pathways Mari *et al*[1]. The bacterium is generally present in the faeces of an infected person for 7 to 14 days, though with treatment, the symptoms do not last long. The bacterium is acquired by humans through eating food or drinking water contaminated by faeces from an infected person. The incubation period of the bacteria is 12 hours to 5 days. During infection the bacteria attach themselves to the intestinal walls where they multiply and produce toxic proteins which cause the intestines to secrete large amounts of fluids. Signs and symptoms include stomach cramps, mild fever, vomiting and watery diarrhoea will lead to death due to dehydration Nelson *et al* [2].

Diagnosis is done through culture of the stool, agglutination tests are then done for confirmation of the disease WHO[3]. The existence of acquired immunity against the cholera disease has been known since very ancient time. Patients recovering cholera are either protected against reinfection with the same *Vibrio cholerae*, or the subsequent episodes are less severe Lavine *et al*[4]. Prevention and control measures of cholera include improved food safety, provision of safe drinking water, proper sanitation, and strengthening surveillance. Health education is also very important Aryda *et al*[5].

Rehydration is critical in reducing cholera death. This has been studied by other scholars but they did not consider the impact of delay in rehydration on the spread of cholera. In this paper, we formulate a mathematical model based on system of ordinary differential equation on the spread of cholera in a logistically growing population to determine the existence of DFE and EE point. For instance Emmanuel *et al.* [6] formulated an SIR-C cholera model to study the dynamics of cholera with control strategy where C denotes the pathogen concentration. Based on their idea, cholera deaths can be reduced by good sanitation and water treatment.

Aryda *et al*[5] developed and analyzed an SIR model to investigate cholera disease with education and chlorination. They concluded that with no chlorination, the disease free equilibrium is shown to be globally stable and the sensitivity analysis of basic reproduction number shows that it is most sensitive to education, per capita birth and death rate of the bacteria. They also concluded that per capita birth and death rate of the bacteria can be increased by chlorination. Owade *et al* [7] developed a cholera model to investigate the role of rehydration and antibiotic treatment on reduction of cholera mortality for individuals with the bacteria in both the intestine and the blood stream. They concluded that rehydration plays a major role in reducing cholera death when the bacteria is in the intestine only and when the bacteria is in both the intestine and the bloodstream it still remains endemic. These models did not consider time delay in rehydration and antibiotic treatment among the cholera patients during cholera outbreak in a logistically growing population.

1.1 The model

The total human population $N(t)$ is divided into classes of susceptible $S(t)$, infected $I(t)$ and recovered $R(t)$. The total human population is given by;

$$N(t) = S(t) + I(t) + R(t) \tag{1}$$

The system of differential equations describing the dynamics of the model is as follows;

$$\begin{aligned} \frac{dS}{dt} &= rS\left(1 - \frac{S}{K}\right) - h(I)S - \mu S \\ \frac{dI}{dt} &= h(I)S - (\mu + \alpha + \beta)I \\ \frac{dR}{dt} &= \alpha I - \mu R \end{aligned} \tag{2}$$

where

$$h(I) = \left[d_1 - \frac{d_2 I(t - \tau)}{q + I(t - \tau)} \right] I \tag{3}$$

1.1.1 The model flow diagram for the dynamics of the transmission

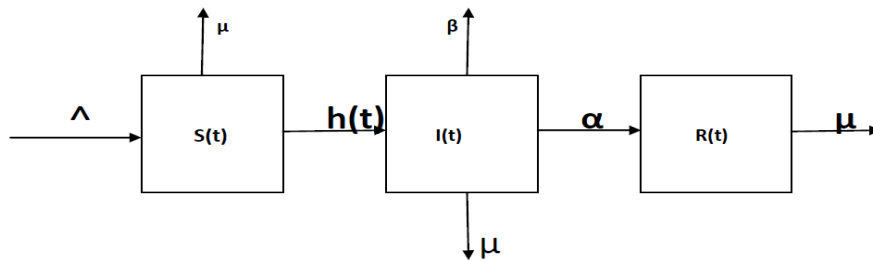


Fig. 1. The model flow diagram

where $\Lambda = rS\left(1 - \frac{S}{K}\right)$. Where $\tau > 0$ is a time delay representing the latent period of rehydration and the term $\frac{d_2 I}{q + I}$ measures the effect of reduction of the contact rate of cholera disease.

In our model, there is a decrease in human population through natural death at a rate μ or as a result of the infection β . Infected individual recover at the rate α and the effect of rehydration given by q

Suppose that the initial condition for the System 2 takes the form:

$$S(t_0) = S(0), I(t_0) = I(0), R(t_0) = R(0); t_0 = 0 \tag{4}$$

2 Model Analysis

2.1 The basic reproduction number, R_0

The basic reproduction number R_0 : Is defined as the average number of secondary infections due to a single infectious individual introduced in a fully susceptible population. If $R_0 < 1$ it means the disease is contained in the population and $R_0 > 1$ means the disease is persistent in the population.

The constant R_0 determined by the method of Next Generation matrix approach Van and Watmough [8] is;

$$R_0 = \frac{d_1 K \left(1 - \frac{\mu}{r}\right)}{\mu + \alpha + \beta}$$

2.2 Existence of Disease Free Equilibrium (DFE) point

Disease Free Equilibrium is defined as the state at which no cholera disease is present in the population.

Proposition 1. For the model system 2, there always exists a DFE point given by $E_0 = (K(1 - \frac{\mu}{r}), 0, 0)$

Proof. At DFE $I = 0$ and $R = 0$, substituting in the first equation of system 2 when $I = 0$ yields; $S_0 = (K(1 - \frac{\mu}{r}))$. Therefore the DFE $E_0 = (K(1 - \frac{\mu}{r}), 0, 0)$ \square

2.3 Existence of endemic equilibrium (EE) points

This is the state where the disease cannot be totally eradicated but remains in the population. This is related to the basic reproduction number R_0

$$R_0 = \frac{d_1 K(1 - \frac{\mu}{r})}{\mu + \alpha + \beta}$$

Theorem 1. Assume that $2d_1d_2 \geq d_1^2 + d_2(r - \mu)$ and $2r\mu \leq 1$ then the model has at least one unique endemic equilibrium $E^*(S^*, I^*, R^*)$ whenever $R_0 > 1$.

Proof. Using the third equation in system 2, $\mu R = \alpha I$

$$R^* = \frac{\alpha I^*}{\mu} > 0, \forall I^* > 0 \tag{5}$$

Using equation two of system 2

$$\begin{aligned} (d_1 - \frac{d_2 I}{q + I})SI &= (\mu + \alpha + \beta)I \\ \frac{(q + I)d_1 - d_2 I}{q + I}S &= (\mu + \alpha + \beta) \\ S^* = \frac{(q + I^*)(\mu + \alpha + \beta)}{(q + I^*)d_1 - d_2 I^*} &> 0, \forall I^* > 0, d_1 > d_2 \end{aligned} \tag{6}$$

Substituting 6 into equation 1 of system 2 at the equilibrium point, we obtain

$$\begin{aligned} r(\frac{(q + I)(\mu + \alpha + \beta)}{(q + I)d_1 - d_2 I})(1 - \frac{1}{K}(\frac{(q + I)(\mu + \alpha + \beta)}{(q + I)d_1 - d_2 I})) \\ - (d_1 - \frac{d_2 I}{q + I})(\frac{(q + I)(\mu + \alpha + \beta)}{(q + I)d_1 - d_2 I})I - \\ \mu(\frac{(q + I)(\mu + \alpha + \beta)}{(q + I)d_1 - d_2 I}) = 0 \end{aligned} \tag{7}$$

Since $S^* \neq 0$ it follows from equation that

$$r(1 - \frac{1}{K}(\frac{(q + I)(\mu + \alpha + \beta)}{(q + I)d_1 - d_2 I})) - (d_1 - \frac{d_2 I}{q + I})I - \mu = 0 \tag{8}$$

Equation 8 can be expressed as

$$AI^2 + BI - C = 0 \tag{9}$$

when solved by help of Matlab we obtain

$$E^* = (S^*, I^*, R^*) = (\frac{(q + I^*)(\mu + \alpha + \beta)}{(q + I^*)d_1 - d_2 I^*}, I^*, \frac{\delta I^*}{\mu}) \tag{10}$$

It can be deduced that $B > 0$, whenever $R_0 > 1$ and the assumption in theorem 1 holds. Equation 10 can therefore be expressed as $-AI^2 + BI - C = 0$ or $AI^2 + BI - C = 0$. In both cases, there exists $I^* > 0$ hence the model has a positive endemic equilibrium. \square

3 Conclusions

I have presented a cholera epidemiological model. There is only environmental to human transmission pathway. The basic reproduction number R_0 plays a crucial role in determining the epidemic and endemic dynamics. For $R_0 < 1$, the disease free equilibrium (DFE) is locally asymptotically stable and unstable for $R_0 > 1$. The disease free equilibrium (DFE) is globally stable when $R_0 < 1$. This means that given any perturbation the disease free equilibrium remains stable. Endemic equilibrium (EE) is also locally asymptotically stable when $R_0 > 1$.

Competing Interests

Authors has declared that no competing interests exist.

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