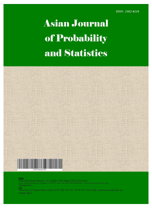

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On the Properties and Applications of a Transmuted Lindley-Exponential Distribution

Adamu Abubakar Umar[1*], Innocent Boyle Eraikhuemen[2], Peter Oluwaseun Koleoso[3], Jerry Joel[4] and Terna Godfrey Ieren[4]

[1]School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia.
[2]Department of Mathematics and Computer Science, Benson Idahosa University, Benin City, Nigeria.
[3]Department of Statistics, University of Ibadan, Ibadan, Oyo State, Nigeria.
[4]Department of Statistics and Operations Research, MAUTech, P.M.B. 2076, Yola, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The Quadratic rank transmutation map proposed for introducing skewness and flexibility into probability models with a single parameter known as the transmuted parameter has been used by several authors and is proven to be useful. This article uses this method to add flexibility to the Lindley-Exponential distribution which results to a new continuous distribution called "transmuted Lindley-Exponential distribution". This paper presents the definition, validation, properties, application and estimation of unknown parameters of the transmuted Lindley-Exponential distribution using the method of maximum likelihood estimation. The new distribution has been applied to a real life dataset on the survival times (in days) of 72 guinea pigs and the result gives good evidence that the transmuted Lindley-Exponential distribution is better than the Lindley-Exponential distribution, Exponential distribution and Lindley distribution based on the dataset used.

Keywords: Quadratic rank transmutation map; Transmuted Lindley-Exponential distribution; definition; properties; maximum likelihood estimation; applications.

**Corresponding author: E-mail: aaumar4real@yahoo.com;*

1 Introduction

An Exponential distribution which can be used in Poisson processes gives a description of the time between events. The distribution has been applied widely in life testing experiments. The distribution exhibits memoryless property with a constant failure rate which makes the distribution unsuitable for real life problems and hence creating a vital problem in statistical modeling and applications.

There are many families of continuous probability distributions useful for adding one or more parameters to a distribution function which makes the resulting distribution more flexible for modeling data. In order to address the problem of memoryless property and constant failure rate of the exponential distribution, many authors have proposed different extensions of the distribution and some of these recent studies on the generalization of exponential distribution include the Lindley-Exponential distribution by Oguntunde et al. [1], the Lomax-exponential distribution by Ieren and Kuhe [2], the transmuted odd generalized exponential-exponential distribution by Abdullahi et al. [3], the transmuted exponential distribution by Owoloko et al, [4], transmuted inverse exponential distribution by Oguntunde and Adejumo [5], the odd generalized exponential-exponential distribution by Maiti and Pramanik [6], the transmuted Weibull-exponential distribution by Yahaya and Ieren [7] and the Weibull-exponential distribution by Oguntunde et al. [8]. To our interest among the aforementioned extensions of the exponential distribution in this article is the Lindley-Exponential distribution which has been found to be useful in various fields for modeling variables whose chances of survival and failure decreases with time (Oguntunde et al. [1]). It has also been discovered that the Lindley-Exponential distribution is positively skewed and performed better than some existing distributions like the conventional Lindley and Exponential distributions (Oguntunde et al. [1]).

The probability density function (pdf) of the Lindley-Exponential distribution according to Oguntunde et al. [1] is defined by:

$$g(x)=\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]\mathrm{e}^{-\alpha\left(\frac{x}{\theta}\right)} \tag{1}$$

The corresponding cumulative distribution function (*cdf*) of Lindley-Exponential distribution is given by:

$$G(x)=1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]\mathrm{e}^{-\alpha\left(\frac{x}{\theta}\right)} \tag{2}$$

“where α is the shape parameter and θ is a scale parameter with, $x>0,\ \theta>0,\ \alpha>0$ ”.

The aim of this paper is to introduce a new continuous distribution called the Transmuted Lindley-Exponential distribution (*TLinExD)* using the proposed quadratic rank transmutation map by Shaw and Buckley [9]. We defined the new distribution with the proof of its validity and its plots in section 2. Section 3 derived some properties of the new distribution. The estimation of parameters using maximum likelihood estimation (MLE) is provided in section 4. An application of the new model with other existing distributions to a real life data is done in section 5 and some useful conclusions are made in section 6.

2 The Transmuted Lindley-Exponential Distribution (*TLinExD*)

2.1 Definition

The *pdf* and *cdf* of the transmuted Lindley-Exponential distribution are defined using the steps proposed by Shaw and Buckley [9]. According to Shaw and Buckley [9], a random variable *X* is said to have a transmuted distribution function if its *pdf* and *cdf* are respectively given by:

$$f(x) = g(x)\left[1+\lambda-2\lambda G(x)\right] \tag{3}$$

and

$$F(x) = (1+\lambda)G(x) - \lambda[G(x)]^2 \tag{4}$$

respectively.

where; x > 0, and $-1 \le \lambda \le 1$ is the transmuted parameter, *G(x)* is the *cdf* of any continuous distribution while *f(x)* and *g(x)* are the associated *pdf* of *F(x)* and *G(x),* respectively.

Substituting equation (1) and (2) in (3) and (4) and simplifying, we obtain the *cdf* and *pdf* of the *TLinExD* as follows:

$$F(x) = (1+\lambda)\left\{1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\} - \lambda\left\{1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}^2,$$

$$F(x) = 1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\} \tag{5}$$

and

$$f(x) = \frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1+\lambda-2\lambda\left\{1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}\right\},$$

$$f(x) = \frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\},$$

$$f(x) = \frac{(1-\lambda)\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)} + \frac{2\lambda\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)} \tag{6}$$

respectively. "where α is the shape parameter, θ is a scale parameter and λ is called the transmuted parameter with, $x > 0$, $\theta > 0$, $\alpha > 0$ and $-1 \le \lambda \le 1$".

2.2 Validity of the model *f(x)*

Recall that for any valid continuous probability distribution, the following integral in (7) must holds, that is

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{7}$$

Proof:

Considering the *pdf* of the transmuted Lindley-Exponential distribution, which is given as

$$f(x)=\frac{(1-\lambda)\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}+\frac{2\lambda\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)}.$$

Substituting the pdf above in equation (7) above, we have:

$$\int_0^\infty f(x)dx=\int_0^\infty\left\{\frac{(1-\lambda)\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}+\frac{2\lambda\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)}\right\}dx$$

$$\int_0^\infty f(x)dx=\int_0^\infty\frac{(1-\lambda)\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}dx+\int_0^\infty\frac{2\lambda\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)}dx$$

$$\int_0^\infty f(x)dx=(1-\lambda)\int_0^\infty\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}dx+\lambda\int_0^\infty\frac{2\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)}dx \tag{8}$$

Recall that from Oguntunde et al. [1], the valid pdf of Lindley-Exponential distribution is given as

$$f(x)=\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}$$

Which implies that

$$\int_0^\infty f(x)dx=\int_0^\infty\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}dx=1.$$

Hence, equation (8) can also be written in the following form

$$\int_0^\infty f(x)dx=(1-\lambda)\int_0^\infty\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}dx+\lambda\int_0^\infty\frac{2\alpha^2}{\theta(\alpha+1)}\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]^2 e^{-2\alpha\left(\frac{x}{\theta}\right)}dx,$$

$$\int_0^\infty f(x)dx=(1-\lambda)(1)+\lambda(1)=1.$$

Therefore,

$$\int_0^\infty f(x)dx=1$$

and we hereby conclude that the pdf the *TLinExD* in equation (6) is a valid probability density function.

2.3 Graphical presentation of Pdf and Cdf of *TLinExD*

The *pdf* and *cdf* of the *TLinExD* using some parameter values are displayed in Figs. 1 and 2 as follows:

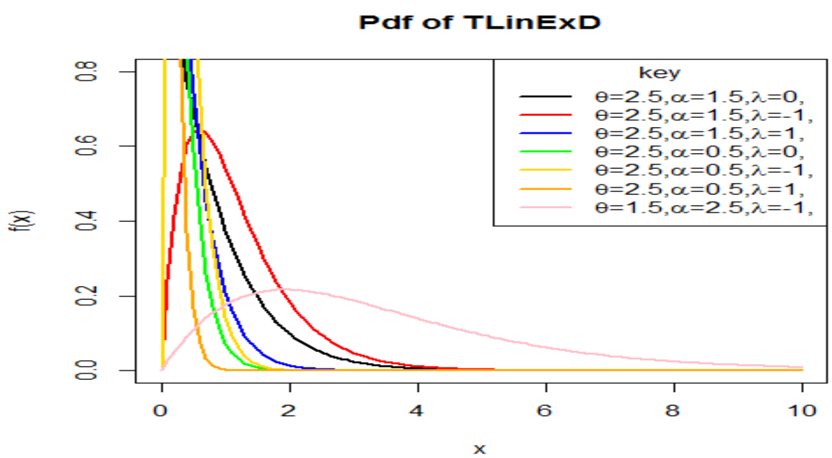

Fig. 1. *PDF* of the *TLinExD* for different values of the parameters

Fig. 1 indicates that the *TLinExD* distribution could be positively skewed and otherwise depending on the parameter values.

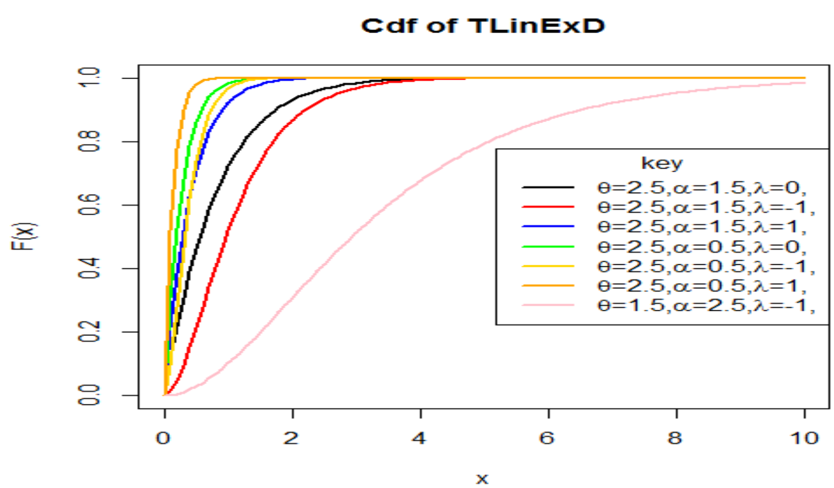

Fig. 2. *CDF* of the *TLinExD* for different values of the parameters

3 Properties

In this section, we defined and discuss some properties of the *TLinExD* distribution.

3.1 Reliability analysis of the *TLinExD*

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \tag{9}$$

Applying the *cdf* of the *TLinExD* in (9), the survival function for the *TLinExD* is obtained as:

$$S(x) = \left[1 + \frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right] e^{-\alpha\left(\frac{x}{\theta}\right)} \left\{1 - \lambda + \lambda\left[1 + \frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right] e^{-\alpha\left(\frac{x}{\theta}\right)}\right\} \tag{10}$$

The following is a plot for the survival function of the *TLinExD* using different parameter values as shown in Fig. 3 below;

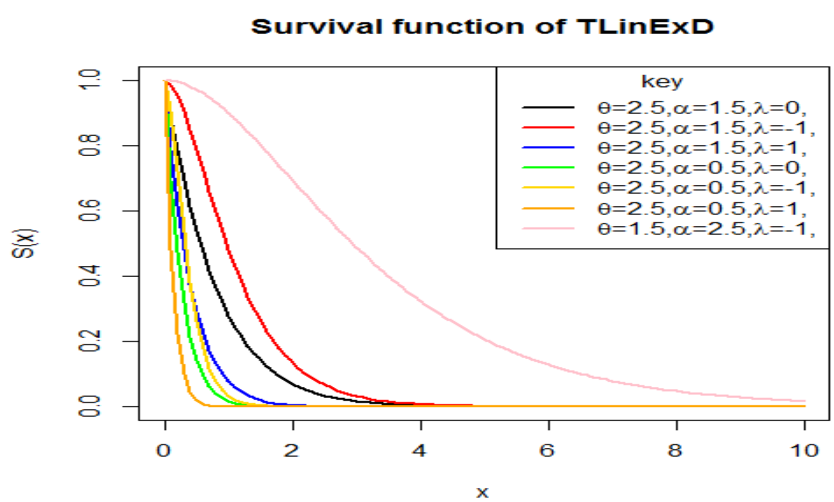

Fig. 3. Survival function of the *TLinExD* at different parameter values

The graph in Fig. 3 shows that the probability of the survival equals one (1) at initial time or early age and it decreases as time increases and equals zero (0) as time becomes larger.

Hazard function is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \tag{11}$$

Meanwhile, the expression for the hazard rate of the *TLinExD* is given by:

$$h(x) = \frac{\alpha^2 \left\{1 - \lambda + 2\lambda\left[1 + \frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right] e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}}{\theta(\alpha+1)\left\{1 - \lambda + \lambda\left[1 + \frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right] e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}} \tag{12}$$

"where α is the shape parameter, θ is a scale parameter and λ is called the transmuted parameter with, $x > 0,\ \theta > 0,\ \alpha > 0$ and $-1 \le \lambda \le 1$".

The following is a plot of the hazard function for arbitrary parameter values in Fig. 4.

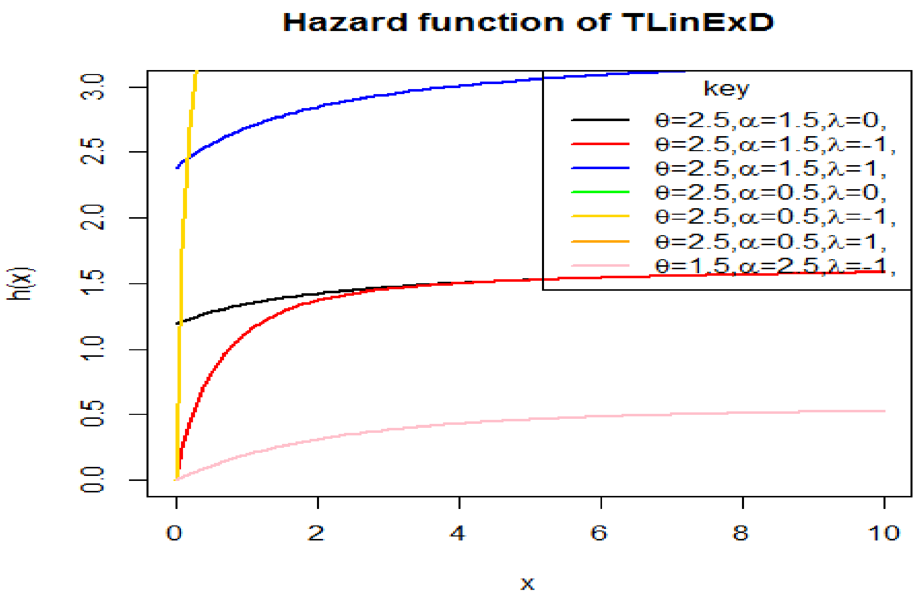

Fig. 4. The hazard function of the *TLinExD* for different values of the parameters as displayed in the key on the graph

Interpretation: the figure above revealed that the *TLinExD* has an increasing failure rate which implies that the probability of failure for any random variable following a *TLinExD* increases.

3.2 Order statistics

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the *TLinExD* and let $X_{1:n}, X_{2:n}, \ldots, X_{i:n}$ denote the corresponding order statistic obtained from this same sample. The *pdf*, $f_{i:n}(x)$ of the i^{th} order statistic can be obtained by:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} \sum_{k=0}^{n-i} (-1)^k \binom{n-i}{k} f(x) F(x)^{k+i-1} \tag{13}$$

Using (5) and (6), the *pdf* of the i^{th} order statistics $X_{i:n}$, can be expressed from (13) as;

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} \sum_{k=0}^{n-i} (-1)^k \binom{n-i}{k} \left\{ \frac{\alpha^2}{\theta(\alpha+1)} \left[1 + \left(\frac{x}{\theta} \right) \right] e^{-\alpha\left(\frac{x}{\theta}\right)} \left\{ 1 - \lambda + 2\lambda \left[1 + \frac{\alpha}{\alpha+1} \left(\frac{x}{\theta} \right) \right] e^{-\alpha\left(\frac{x}{\theta}\right)} \right\} \right\}$$
$$\times \left\{ 1 - \left[1 + \frac{\alpha}{\alpha+1} \left(\frac{x}{\theta} \right) \right] e^{-\alpha\left(\frac{x}{\theta}\right)} \left\{ 1 - \lambda + \lambda \left[1 + \frac{\alpha}{\alpha+1} \left(\frac{x}{\theta} \right) \right] e^{-\alpha\left(\frac{x}{\theta}\right)} \right\} \right\}^{i+k-1} \tag{14}$$

Hence, the *pdf* of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the *TLinExD* are respectively given by;

$$f_{1n}(x)=n\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\left\{\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}\right\}$$
$$\times\left\{1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}\right\}^k \quad (15)$$

and

$$f_{nn}(x)=n\left\{\frac{\alpha^2}{\theta(\alpha+1)}\left[1+\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}\right\}$$
$$\times\left\{1-\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\left\{1-\lambda+\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x}{\theta}\right)\right]e^{-\alpha\left(\frac{x}{\theta}\right)}\right\}\right\}^{n-1} \quad (16)$$

4 Estimation of Unknown Parameters of the *TLinExD*

Let $X_1, X_2, ..., X_n$ be a sample of size 'n' independently and identically distributed random variables from the *TLinExD* with unknown parameters *α, θ* and *λ* defined previously. The likelihood function is given by;

$$L\left(\underline{X}/\alpha,\theta,\lambda\right)=\frac{\alpha^{2n}}{\theta^n(\alpha+1)^n}\prod_{i=1}^{n}\left(\left(1+\left(\frac{x_i}{\theta}\right)\right)e^{-\alpha\left(\frac{x_i}{\theta}\right)}\right)\prod_{i=1}^{n}\left[1-\lambda+2\lambda\left(1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right)e^{-\alpha\left(\frac{x_i}{\theta}\right)}\right] \quad (17)$$

Let the natural logarithm of the likelihood function be, $l(\eta)=\log L\left(\underline{X}\mid\alpha,\beta,\theta,\lambda\right)$, therefore, taking the natural logarithm of the function above gives:

$$l(\eta)=2n\log\alpha-n\log\theta-n\log(\alpha+1)+\sum_{i=1}^{n}\log\left(1+\left(\frac{x_i}{\theta}\right)\right)-\alpha\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)+\sum_{i=1}^{n}\log\left[1-\lambda+2\lambda\left(1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right)e^{-\alpha\left(\frac{x_i}{\theta}\right)}\right] \quad (18)$$

Differentiating $l(\eta)$ partially with respect to *α, θ* and *λ* respectively gives the following results;

$$\frac{\partial l(\eta)}{\partial\alpha}=\frac{2n}{\alpha}-\frac{n}{\alpha+1}-\frac{1}{\theta}\sum_{i=1}^{n}x_i-\frac{2\lambda}{\theta}\sum_{i=1}^{n}\left\{\frac{x_i\left[\frac{1}{(\alpha+1)^2}-\frac{\alpha x_i}{\theta(\alpha+1)}-1\right]e^{-\alpha\left(\frac{x_i}{\theta}\right)}}{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right]e^{-\alpha\left(\frac{x_i}{\theta}\right)}}\right\} \quad (19)$$

$$\frac{\partial l(\eta)}{\partial \theta}=-\frac{n}{\theta}-\frac{1}{\theta^{2}}\sum_{i=1}^{n}\left\{\frac{x_i}{\left(1+\theta^{-1}x_i\right)}\right\}+\frac{\alpha}{\theta^{2}}\sum_{i=1}^{n}x_i+\frac{2\alpha\lambda}{\theta^{2}}\sum_{i=1}^{n}\left\{\frac{x_i\left[\frac{\alpha x_i}{\theta(\alpha+1)}-\frac{1}{(\alpha+1)}+1\right]\mathrm{e}^{-\alpha\left(\frac{x_i}{\theta}\right)}}{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right]\mathrm{e}^{-\alpha\left(\frac{x_i}{\theta}\right)}}\right\} \tag{20}$$

$$\frac{\partial l(\eta)}{\partial \lambda}=\sum_{i=1}^{n}\left\{\frac{2\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right]\mathrm{e}^{-\alpha\left(\frac{x_i}{\theta}\right)}-1}{1-\lambda+2\lambda\left[1+\frac{\alpha}{\alpha+1}\left(\frac{x_i}{\theta}\right)\right]\mathrm{e}^{-\alpha\left(\frac{x_i}{\theta}\right)}}\right\} \tag{21}$$

The solution of the non-linear system of equations of $\frac{\partial l(\eta)}{\partial \alpha}=0,\ \frac{\partial l(\eta)}{\partial \theta}=0$ and $\frac{\partial l(\eta)}{\partial \lambda}=0$ gives the maximum likelihood estimates of parameters α, θ and λ. Meanwhile, the solution cannot be obtained analytically except numerically with the aid of suitable statistical software like *R*, *SAS*, *MATHEMATICA e.t.c* and therefore a dataset is considered in the next section to fit the proposed distribution.with other distributions using "maxLik" package in R software.

5 Application to a Real Life Dataset

This section presents a real life dataset, its descriptive statistics, graphical summary and applications of the transmuted Lindley-Exponential distribution (*TLinExD*), Lindley-Exponential distribution (LinExD), Exponential distribution (ExD) and Lindley distribution (LinD) to the dataset.

In order to evaluate the efficiency of the models mentioned above, a package named "maxLik" was used in R Statistical software environment developed by R Core Team [10] with method "SANN". The result of the analysis presents estimates of the model parameters as well as the value of the log-likelihood function and we computed a model selection criterion called Akaike Information Criterion, AIC. The formula for this statistic is given as:

$$AIC = -2ll + 2k\ ,$$

"where ll denotes the value of the log-likelihood evaluated at the *MLEs*, k is the number of parameters in the distribution.

Hence, the distribution with the smallest value of AIC will be considered as the best model to fit the data.

Dataset: The dataset represents the survival times (in days) of 72 guinea pigs infected with virulenttubercle bacilli reported by Bjerkedal [11]. They are the Regiment 4.3, Study M.: 10, 33, 44,56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116,120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176,183, 195, 196, 197, 202, 213, 215, 216, 222, 230,231, 240, 245, 251, 253, 254, 255, 278, 293, 327,342, 347, 361, 402, 432, 458, 555.

From the descriptive statistics in Table 1 and the histogram, box plot, density and normal Q-Q plot shown in Fig. 5, we observed that dataset on the survival times (in days) of 72 guinea pigs is positively skewed.

Table 1. Descriptive Statistics for the survival times (in days) of 72 guinea pigs data

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Dataset I	72	10.0	108.0	149.5	224.0	176.8	555.0	10705.1	1.34128	1.98852

Table 2. Performance of the fitted distributions using the value of AIC based on the dataset

Distributions	Parameter estimates	Log-likelihood value	*AIC*	Rank of models
TLinExD	$\hat{\alpha} =$ 0.1404117	425.4224	856.8448	1st
	$\hat{\theta} =$ 9.3727146			
	$\hat{\lambda} =$ -0.8954846			
LinExD	$\hat{\alpha} =$ 0.1065785	433.1593	870.3185	3rd
	$\hat{\theta} =$ 8.1663538			
ExD	$\hat{\theta} =$ 0.004447419	446.5356	895.0712	4th
LinD	$\hat{\alpha} =$ 0.01124718	429.2848	860.5697	2nd

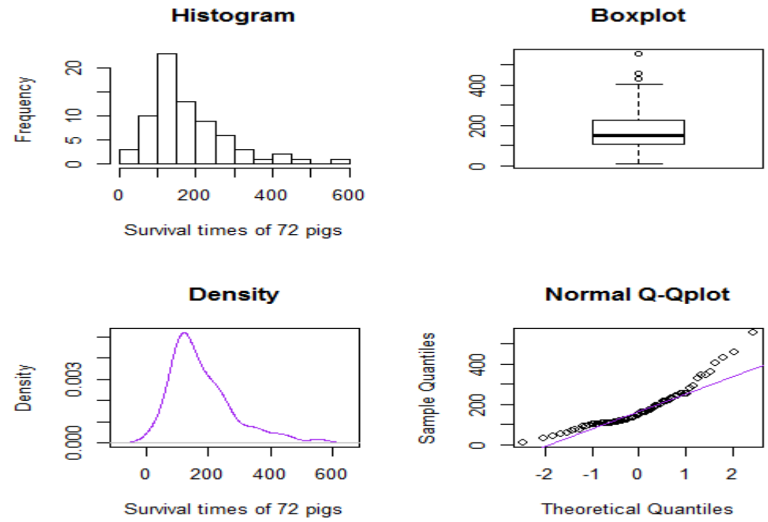

Fig. 5. A graphical summary of the survival times (in days) of 72 guinea pigs data

The following figures displayed the histogram and estimated densities and cdfs of the fitted models to the survival times (in days) of 72 guinea pigs data.

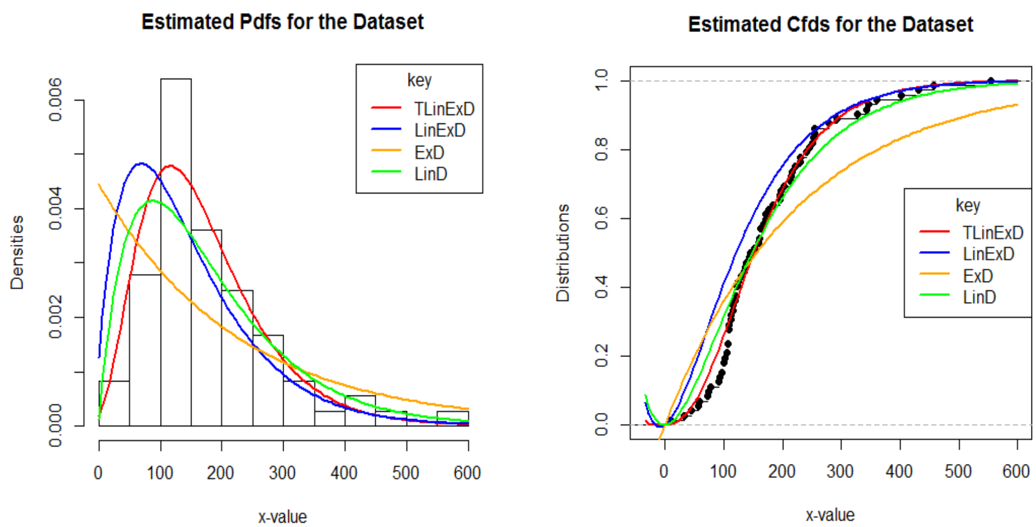

Fig. 6. Histogram and plots of the estimated densities and cdfs of the fitted distributions to the survival times (in days) of 72 guinea pigs dataset

Table 2 presents the parameter estimates and the values of AIC for the the *TLinExD*, LinExD, LinD and ExD using dataset which is skewed to the right. The values of AIC in Table 2 reveal that the transmuted Lindley-Exponential distribution (*TLinExD*) is better than the LinExD, LinD and ExD and this performance is also demonstrated in the estimated density plots in Fig. 6 as well as the Q-Q plots presented in Fig. 7. This result confirms the that fact that the quadratic rank transmutation map by Shaw and Buckley [9] has additional advantage to the Lindley-Exponential distribution by increasing its skewness and flexibility in modeling real life data and therefore, we agree and conclude that the quadratic rank transmutation map proposed by Shaw and Buckley [9] is very useful in increasing the flexibility of continuous probability distributions as seen in the studies of Merovci [12], Merovci and Puka [13], Yahaya and Ieren [7], Abdullahi and Ieren [14], Abdullahi et al. [3], Oguntunde and Adejumo [5] and Owokolo et al. [4] and so on.

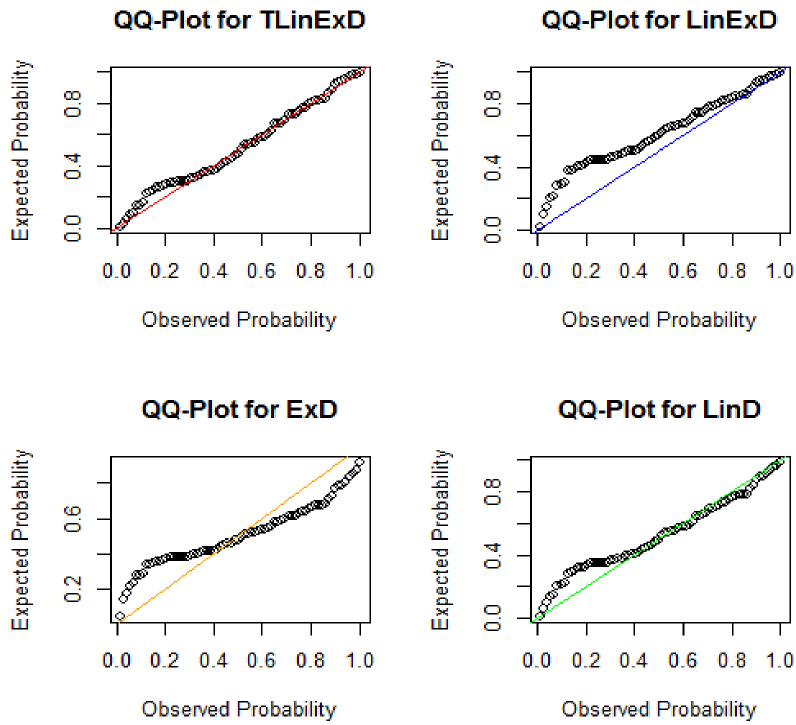

Fig. 7. Probability plots for the fit of the *TLinExD*, LinExD, LinD & ExD based on the survival times (in days) of 72 guinea pigs dataset

6 Summary and Conclusion

This study proposed a new three-parameter extension of the Exponential distribution named "Transmuted Lindley-Exponential distribution". A very popular reason for generalizing a classical distribution is the fact that the generalization makes it more flexible for analyzing real life data. In this paper, some of mathematical and statistical properties of the transmuted Lindley-Exponential distribution are derived and studied. The paper also obtained the density function for the distribution of minimum and maximum order statistics. It also estimates the model parameters by method of maximum likelihood estimation. The new distribution has been applied to a real life dataset on the survival times (in days) of 72 guinea pigs and the results give evidence that the transmuted Lindley-Exponential distribution is better than the Lindley-Exponential distribution, Exponential distribution and Lindley distribution based on the dataset used.

Competing Interests

Authors have declared that no competing interests exist.

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